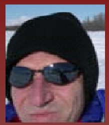
Curved Beam Model of Mandibular Symphyseal Bending including Heterogeneous Elasticity



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Overview

Comparative biomechanical study of the anthropoid mandibular symphysis proceeds from the premise that it behaves as a curved beam with respect to bending due to laterally directed components of masticatory muscle forces. Such a model has been used to explore functional linkages between mandibular form and masticatory loads. This "strength of materials" approach to characterize symphyseal stress assumes the midsagittal section of the symphysis to be an idealized geometric shape under symmetrical bending with a single elastic modulus throughout (i.e., homogeneous elasticity). We improve upon this model by including material heterogeneity inherent in bone via an elastic modulus distribution that reflects an experimentally determined variation. Predicted strains are validated with full-field strains measured by speckle image photogrammetry. Our model and experimental results indicate that the characteristic inclination of the symphysis functions to alleviate the potential increase in strain in the inferior transverse torus caused by the combined effects of a curved mandibular arch and lingually compliant bone.

The Model

Determine* orientation of cross-sectional weighted principal centroidal axes (*ca*) *y* & *z*

Resolve wishboning moment (from masticatory muscle force *F*) into components M_y & M_z

Determine* normalized weighted moments of inertia I_y & I_z

Measure symphyseal radius of curvature *R*, length 2*c* & width *b*

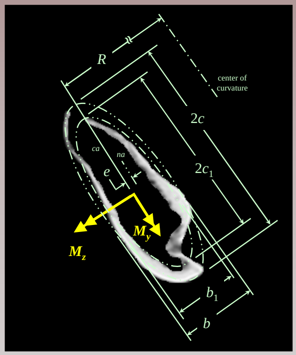
Assume similar-elliptical tube of outer diameters 2*c* & *b* such that inner diameters $b_1 c = b c_1$

Equate I_z to that of similar-elliptical tube $I_{\text{tube}} = \frac{\pi}{8}\left(bc^3 - b_1 c_1^3\right)$ so that $c_1 = \left(c^4 - \frac{8cI_z}{\pi b}\right)^{1/4}$

Compute† eccentricity *e* of neutral axis (*na*), i.e., distance between *na* & *ca* from

$$e = R - \frac{\left(c - \frac{b_1 c_1}{b}\right)}{2\left\{\frac{R}{c} - \sqrt{\left(\frac{R}{c}\right)^2 - 1} - \frac{b_1 c}{b c_1}\left[\frac{R}{c} - \sqrt{\left(\frac{R}{c}\right)^2 - \left(\frac{c_1}{c}\right)^2}\right]\right\}}$$

Determine† normal stresses, e.g., $\sigma_{\text{labial, lingual}} = \frac{F}{A_{\text{tube}}} + \beta_{\text{labial, lingual}} \frac{M_y b}{2 I_y}$ where $\beta_{\text{labial }(+),\text{ lingual }(-)} = \frac{c\left(1 \pm \frac{e}{c}\right)\left[1 - \frac{b_1}{b}\left(\frac{c_1}{c}\right)^3\right]}{4e\left(\frac{R}{c} \pm 1\right)\left(1 - \frac{b_1 c_1}{bc}\right)}$



* Bhatavadekar, Daegling & Rapoff *AJPA* 2006
† Cook & Young *Advanced Mechanics of Materials* 1999

Validation

Subject mandible to "wishbone" loading in materials testing machine

Measure full-field surface strains using photogrammetry

Use regional elastic moduli‡ & Hooke's law to determine normal strains

Compare predicted & measured strains

‡ Daegling, Hotzman, McGraw & Rapoff *AJPA* 2007