MER 440 ORTHOPEDIC BIOMECHANICS

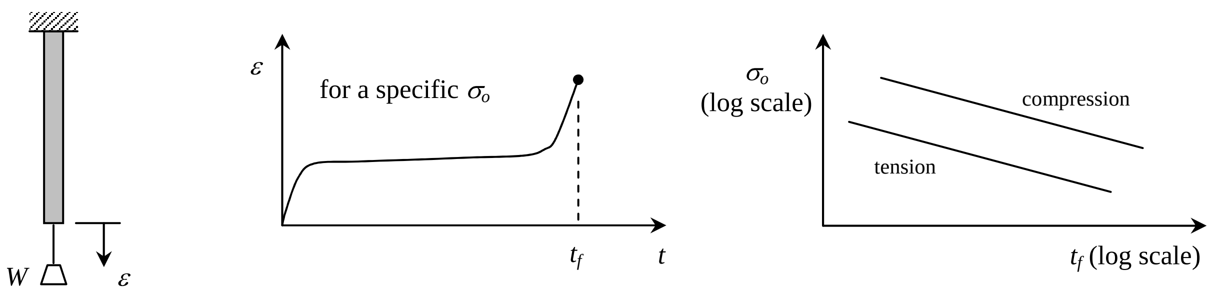
CREEP & FATIGUE DAMAGE IN BONE

Creep

Bone is a viscoelastic material that exhibits creep behavior, i.e., increasing deformation under the action of a constant load. If a static weight W, causing a stress σ_o, is hung from the end of a bone specimen, the specimen will eventually break after a time to failure t_f. If many different stress magnitudes are investigated, a plot of σ_o versus t_f can be made, and, for bone, such a plot reveals a linear relationship in log-log space that can be represented by

$$t_f = A\sigma_o^{-B}$$

where A and B are empirical constants. These constants have been determined (Caler & Carter *J Biomech* 1989) to be $A = 3.02 \times 10^{25}$ s and $B = 17.95$ if the units on stress are [MPa]. The constant B is similar and A is much greater for compressive creep. For example, the time to failure under a constant 20 MPa stress is over 400 centuries and for 60 MPa about 1 hour.



Damage

A dictionary definition of damage includes "[physical injury that makes something less able to function]". Physical damage in bone is in the form of cracks. Damage D in an engineering context refers to a quantitative measure of the effect of the physical damage on mechanical performance. In this way, a physically undamaged structure is represented by $D = 0$, and a failed structure is represented by $D = 1$. As an example, Miner's rule is used to predict fatigue failure for structures subjected to multiple constant amplitude blocks of fatigue loading at various amplitudes (i.e., a restricted sort of variable amplitude loading) or for structures subjected to completely variable amplitude loading. In Miner's rule, damage is defined as

$$D = \sum_{i=1}^{\text{\# of blocks}} \frac{n_i}{N_i}$$

where N_i is the fatigue life for a stress S_i and n_i is the number of cycles of applied stress S_i. If the loading is completely variable in amplitude, the "cycles" is replaced by "intervals" and a characteristic stress level is used.

Damage in bone can be decomposed into damage including that due to mechanical loading (D_σ), aging (D_A), disease (D_D), exercise (D_E), remodeling (D_R), pharmaceuticals (D_P), ..., and interactions between all of these modes (D_I, and itself maybe pair-wise decomposed). The total damage is then given by

$$D = D_\sigma + D_A + D_D + D_E + D_R + D_p + \cdots + D_I$$

Note that "reparative" damages, e.g., due to healing, would be negative. The mechanical damage in bone can be decomposed into that due to fatigue and creep such that

$$D_\sigma = D_{\text{fatigue}} + D_{\text{creep}}$$

The damage due to fatigue can be found from the *S-N* power law expression $\Delta\sigma = \sigma_u N^{-m}$ and by noting that the time to failure under fatigue loading is $t_f = N/f$ where f is the frequency of loading. Also note that the stress range $\Delta\sigma$ is used in the expression, which could represent some characteristic value of the stress range for non-constant amplitude loading. Substituting and solving yields

$$\frac{1}{t_f} = \left(\frac{\Delta\sigma}{\sigma_u}\right)^{1/m} f$$

The simplest assumption is that the fatigue damage evolves at a constant rate, i.e., the frequency of damage evolution is a constant such that

$$\frac{dD_{\text{fatigue}}}{dt} = \frac{1}{t_f} = \left(\frac{\Delta\sigma}{\sigma_u}\right)^{1/m} f$$

Integrating from zero damage at time zero to some damage at some later time yields

$$D_{\text{fatigue}} = \left(\frac{\Delta\sigma}{\sigma_u}\right)^{1/m} ft$$

Similarly, if the damage due to creep evolves at a constant rate such that

$$\frac{dD_{\text{creep}}}{dt} = \frac{1}{t_f} = \frac{1}{A}\left[\sigma(t)\right]^B$$

where the stress is now time-varying. For example, if the loading is an all tension sine wave such that the minimum stress is zero (i.e., $R = 0$), the time varying stress is given by

$$\sigma(t) = \frac{\Delta\sigma}{2}\left[1 + \sin\left(2\pi ft - \frac{\pi}{2}\right)\right]$$

Note that the non-zero mean stress present with some time-varying stresses can be thought of as the "constant stress" that leads to the creep damage. Also note that for lower loading frequencies, non-zero stresses are applied for a longer period of time for the same number of cycles compared to higher loading frequencies (e.g., 100 s at 1 Hz versus 10 s at 10 Hz for 100 cycles of loading). Finally, note that laboratory fatigue tests of bone necessarily involve high stresses applied at high frequencies so that tests are completed in a reasonable length of time

For the general time-varying stress case, integrating from zero damage at time zero to some damage at some later time yields

$$D_{\text{creep}} = \frac{1}{A}\int_0^t \left[\sigma(\tau)\right]^B d\tau$$

The damage due to fatigue and creep can be re-combined

$$\boxed{D_\sigma = \left(\frac{\Delta\sigma}{\sigma_u}\right)^{1/m} ft + \frac{1}{A}\int_0^t \left[\sigma(\tau)\right]^B d\tau}$$

<u>Relationship to *In Vivo* Loading</u>

The above expression for mechanical damage is now used to determine what damage mode is dominant under *in vivo* loading conditions. Fairly involved plots (from Carter & Caler *J Orthop Res* 1985) follow that are worthwhile to study until understood. In this discussion, the term "model" refers to the above expression.

In the top plot, the experimental data consists of fully reversed (**TENSION-COMPRESSION** solid circles) and **ZERO-TENSION** (open circles) conducted on normal adult human cortical bone specimens at 2 Hz. At first, a *S-N* curve of the data was plotted, i.e., stress range $\Delta\sigma$ vs. cycles to failure *N*. The elastic strain range $\Delta\varepsilon$ was computed using Hooke's law with a longitudinal Young's modulus of 17 GPa to construct the strain range $\Delta\varepsilon$ scale at right. The time to failure $t_f = N/f$ was computed knowing the frequency of loading to construct the time scale at top. The dashed **FATIGUE DAMAGE** line represents the case when $D_{\text{creep}} = 0$ and $D_{\text{fatigue}} = 1$ applies, for fully-reversed cyclic loading, and the model fits those data well (the model at least "splits" the bottom three solid circles). The dashed **CREEP DAMAGE** line represents the case when $D_{\text{creep}} = 1$ and $D_{\text{fatigue}} = 0$ applies. We know that this applies to non-fully-reversed cyclic loading, and the model appears to diverge from the last two open circle points on the right. This divergence indicates that in this low stress range regime, assuming $D_{\text{creep}} = 1$ and $D_{\text{fatigue}} = 0$ is incorrect: $D_{\text{fatigue}} \neq 0$ contributes to D_σ as well and is represented by the solid **MODEL PREDICTION** curve.

In the bottom plot, the relative contributions (**DAMAGE FRACTION**) are depicted of D_{creep} and $D_{fatigue}$ to D_{σ} for the **ZERO-TENSION** cyclic loading solid **MODEL PREDICTION** curve of the top plot. Note that in the high stress regime, D_{creep} dominates, while $D_{fatigue}$ dominates in the **NORMAL** physiologic low stress regime. Therefore, the dominant damage mode switches from fatigue to creep at a stress range of about 60 MPa. Most *in vivo* loading involves stresses much less severe such that fatigue is the dominant damage mode.