MATERIAL SYMMETRY

Any symmetry in material structure will be reflected by symmetry in mechanical (and other) properties. Symmetry, or the lack thereof, can be defined at a single material particle and/or within a region of material and can change from particle-to-particle and/or region-to-region. An inhomogeneous material results if the symmetry changes and/or the properties change throughout the material.

ANISOTROPIC MATERIAL

An anisotropic material lacks any material symmetry. The elasticity tensor (or tensor of elastic constants) C^*_{ijkl}, that relates stresses to tensor strains, is fully populated and contains 81 terms. However, due to symmetry of the stress and strain tensors and due to the existence of the strain energy function, the number of independent elastic constants is reduced to 21 for the fully anisotropic material.

$$C_{ij}\Big|_{\text{anisotropic}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}$$

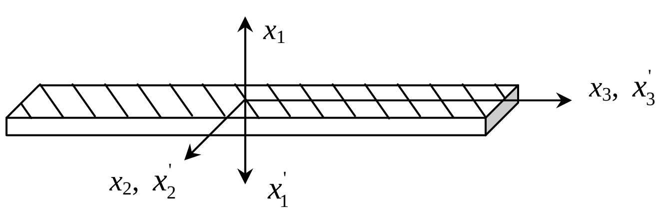
Some single crystal materials are anisotropic, but this complete lack of symmetry in a material is rarely encountered.

MONOCLINIC MATERIAL

A monoclinic material is symmetric about a single plane and possesses 13 independent elastic constants. This can be shown by performing a specific coordinate transformation and enforcing the fact that the elastic constants must not change (i.e., the elasticity tensor is invariant to the transformation). For example, as shown in the below figure, if the plane of symmetry is the x_2-x_3 plane, then the transformation

$$C^{*'}_{ijkl} = a_{mi} a_{nj} a_{rk} a_{sl} C^*_{mnrs}$$

to the x_i' system shown must result in no change in the elastic constants.



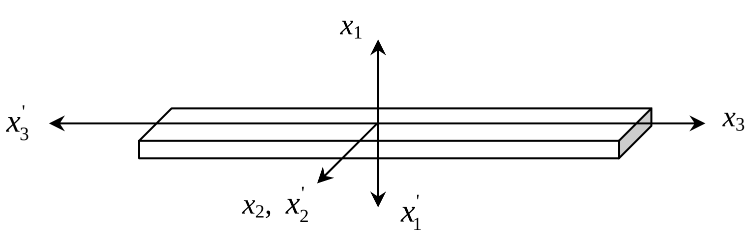
From this transformation, 8 of the 21 fully anisotropic constants must vanish, and the matrix of elastic constants becomes

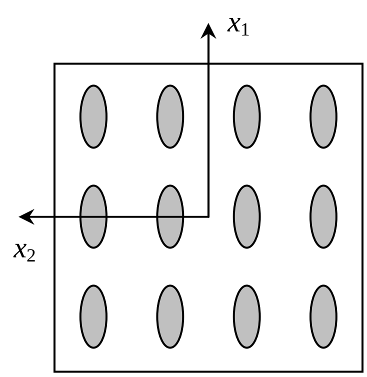
$$C_{ij}\Big|_{\substack{\text{monoclinic} \\ \text{symmetric about } x_2-x_3 \text{ plane}}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0 \\ C_{13} & C_{23} & C_{33} & C_{34} & 0 & 0 \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

Note that the new coordinate system is not "right handed," thus the transformation is not a proper orthogonal transformation. An example of a monoclinic material is the so-called off-axis unidirectional fiber composite lamina. In the figure, the cross-hatching lines are meant to indicate the fiber orientations.

ORTHOTROPIC MATERIAL

An orthotropic material is symmetric about three orthogonal planes and possesses 9 independent elastic constants. This can be shown by performing a 180° rotation of two of the coordinate axes as shown in the figure at left and enforcing no change in the elastic constants. An example of an orthotropic material is the so-called on-axis unidirectional fiber composite lamina where the fibers are elliptical in cross section. In the figure at left, the fibers would be aligned with the x_3 axis, and their elliptical cross section are shown in the figure at right.



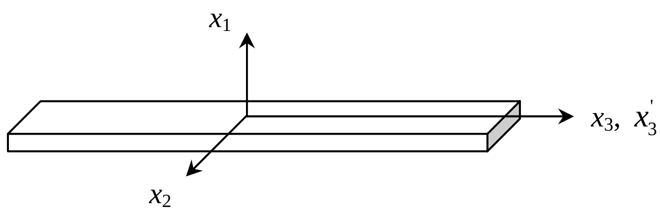


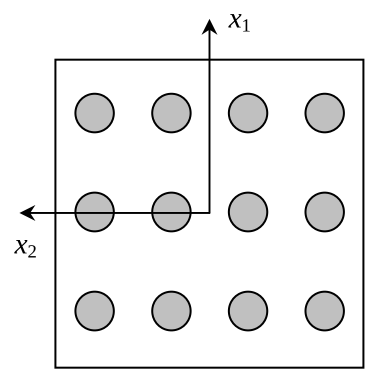
The matrix of elastic constants is

$$C_{ij}\Big|_{\text{orthotropic}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

TRANSVERSELY ISOTROPIC MATERIAL

A transversely isotropic material is symmetric about an axis that is normal to a plane of isotropy and possesses 5 independent elastic constants. This can be shown by performing transformations involving any rotation about the x_3 axis as shown in the figure at left and enforcing no change in the elastic constants. An example of a transversely isotropic material is the so-called on-axis unidirectional fiber composite lamina where the fibers are circular in cross section. In the figure at left, the fibers would be aligned with the x_3 axis, and their circular cross section are shown in the figure at right.





The matrix of elastic constants is

$$C_{ij}\Big|_{\substack{\text{transversely isotropic}\\ \text{symmetric about } x_3 \text{ axis}}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{C_{11}-C_{12}}{2} \end{bmatrix}$$

ISOTROPIC MATERIAL

An isotropic material possesses an infinite number of planes of symmetry and only 2 independent elastic constants. This can be shown by performing any transformation and enforcing no change in the elastic constants, which are given by

$$C_{ij}\Big|_{\text{isotropic}} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{(C_{11}-C_{12})}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{(C_{11}-C_{12})}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{(C_{11}-C_{12})}{2} \end{bmatrix}$$

Many materials are assumed to be isotropic.